Keyword Spotting with Quaternionic ResNet: Application to Spotting in Greek Manuscripts

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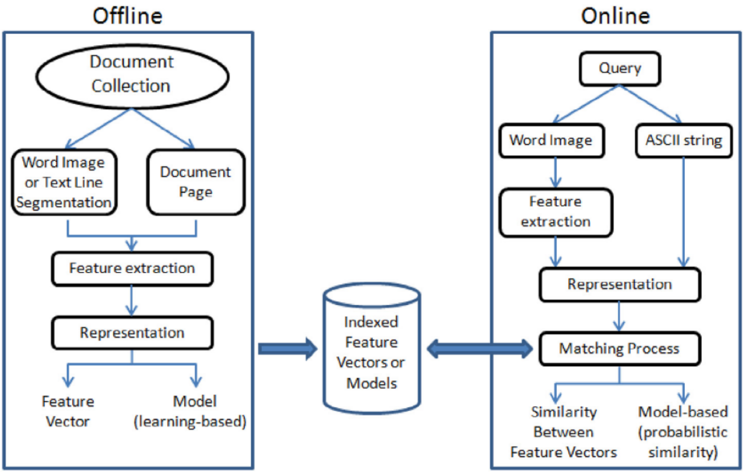
Keyword spotting



Our contribution in a nutshell

- Introduction of a Quaternionic version of the ResNet block
- Testing on KWS
- **Result: As effective as vanilla version but much "lighter"**

Basic form

Quaternions $q \in \mathbb{H}$ share the basic form:

$$q = a + b\boldsymbol{i} + c\boldsymbol{j} + d\boldsymbol{k}$$

where $a, b, c, d \in \mathbb{R}$ and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are independent imaginary units.

Generalizations of complex numbers

- Two ways to see this:
  - 2 extra imaginary units $(\boldsymbol{j}, \boldsymbol{k})$. Real and complex numbers can be regarded as quaternions with $b, c, d = 0$ or $c, d = 0$ respectively.
  - OR: $a + b\boldsymbol{j}$ with $a, b \in \mathbb{C}$ (Cayley-Dickson form)

Historical note

- William Rowan Hamilton (1805-1865) discovered quaternions, initially in his search of a way to multiply triplets (elements of $\mathbb{R}^3$)
- He discovered that a fourth element is "necessary" ($\mathbb{R}, \mathbb{C}$ and $\mathbb{H}$ are the only possible commutative division algebras [Frobenius, 1878])

Basic "peculiarity": Multiplication is non-commutative

$$pq \neq qp \;\text{ for }\; p, q \in \mathbb{H}$$

Algebraic structure

Quaternions are mathematical objects that form a skew-field $\mathbb{H}$, *i.e.* quaternion addition and multiplication are defined with all the properties of a field, except that of multiplication commutativity

Multiplication rule (Hamilton product)

$$
\begin{aligned}
pq = &(a_p a_q - b_p b_q - c_p c_q - d_p d_q) + &(1)\\
&(a_p b_q + b_p a_q + c_p d_q - d_p c_q)\boldsymbol{i} + &(2)\\
&(a_p c_q - b_p d_q + c_p a_q + d_p b_q)\boldsymbol{j} + &(3)\\
&(a_p d_q + b_p c_q - c_p b_q + d_p a_q)\boldsymbol{k}, &(4)
\end{aligned}
$$

where $p = a_p + b_p\boldsymbol{i} + c_p\boldsymbol{j} + d_p\boldsymbol{k}$ and $q = a_q + b_q\boldsymbol{i} + c_q\boldsymbol{j} + d_q\boldsymbol{k}$.

We can write the above rule also as:

$$pq = S(p)S(q) - V(p)\cdot V(q) + S(p)V(q) + S(q)V(p) + V(p)\times V(q), \quad (5)$$

Properties

Other elementary relations

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1$$

$$\boldsymbol{ij} = -\boldsymbol{ji} = \boldsymbol{k}, \boldsymbol{jk} = -\boldsymbol{kj} = \boldsymbol{i}, \boldsymbol{ki} = -\boldsymbol{ik} = \boldsymbol{j}$$

$$|q| = \sqrt{q\bar{q}} = \sqrt{\bar{q}q} = \sqrt{a^2 + b^2 + c^2 + d^2}$$

$$\bar{q} = a - b\boldsymbol{i} - c\boldsymbol{j} - d\boldsymbol{k}$$

Caley-Dickson form

$$q = \zeta + \eta \boldsymbol{j}$$

$$\zeta = a + b\boldsymbol{i}, \eta = c + d\boldsymbol{i}.$$

Properties

Euler's identity

Euler's identity extends for quaternions, with μ being an arbitrary unit pure quaternion:

$$e^{\mu\theta} = cos\theta + \mu \sin\theta.$$

Convolution

Left-Right convolution

$$(h_L * f)[n,m] = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} h_L[i,j] f[n-i, m-j], \tag{6}$$

$$(f * h_R)[n,m] = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} f[n-i, m-j] h_R[i,j], \tag{7}$$

Biconvolution

$$(h_L \prec f \succ h_R)[n,m] = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} h_L[i,j] f[n-i, m-j] h_R[i,j]. \tag{8}$$

In Neural Networks

- *Dense layer*

$$f_{dense}(x; W, h) = Wx + h$$

- *Convolution layer*

$$f_{conv}(m; K) = K * m$$

- *ResNet block*

$$f_{ResBlock} = \alpha(\phi(x) + x)$$

Pros and cons of quaternions

Argumentum ab auctoritate

Quaternions have had proponents for their use as well as against since their inception

The arguments against quaternions..

- $\mathbb{H}$ is non-commutative, which creates all sorts of problems
- Set $\mathbb{H}$ is isomorphic to $\mathbb{R}^4$, which is less complex and better understood (so a quaternion can be replaced by a 4-dimensional vector)

Pros and cons of quaternions

..so why should we bother?

+ In the context of Neural Networks, they lead to "light-weight" layer defitions (=requiring much less parameters)

Why are quaternionic layers less costly?

Let us illustrate this with a simple example. Assume a real-valued linear layer, without bias or activation, that transforms an input comprised of 4 neurons to an output of 4 neurons.

- $x \in \mathbb{R}^4$ is mapped to an output $y \in \mathbb{R}^4$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \alpha & \beta & \gamma & \delta \\ \epsilon & \zeta & \eta & \theta \\ \iota & \kappa & \lambda & \mu \\ \nu & \xi & o & \pi \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \tag{9}$$

Why are quaternionic layers less costly?

- Quaternionic version of the same operation

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \alpha & -\beta & -\gamma & -\delta \\ \beta & \alpha & -\delta & \gamma \\ \gamma & \delta & \alpha & -\beta \\ \delta & -\gamma & \beta & \alpha \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \tag{10}$$

- Corresponds to Hamilton product $y = wx$
- This paradigm extends to cover any-size inputs and outputs (as long as they are multiples of 4)

Architecture

- QNN defined as a cascade of Quaternion ResNet blocks
- On each block: convolution (stride=1, kernel size=3×3) $\rightarrow$ BN $\rightarrow$ ReLU...
- ...$\rightarrow$ convolution $\rightarrow$ BN (function ϕ)
- Whole block: $ReLU(\phi(x) + x)$
- 7 blocks in total
- "Split-activation" sigmoid activation maps output to PHOC estimates

Datasets

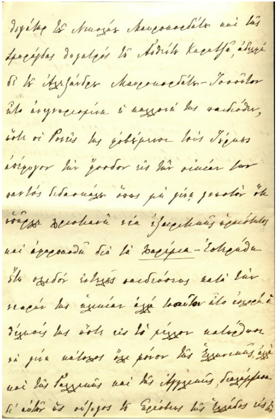
Memoirs

46 manuscripts - segmented into a total of $4,941$ words

PIOP-DAS

22 manuscripts - segmented into a total of $12,362$ words

Datasets



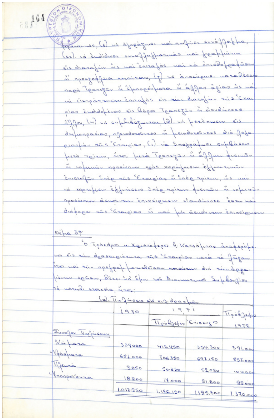


Figure: Sample pages from the two datasets used in this work, "Memoirs" and "PIOP-DAS".

Sample word images

Figure: Word image samples from the “PIOP-DAS” dataset.

Sample word images

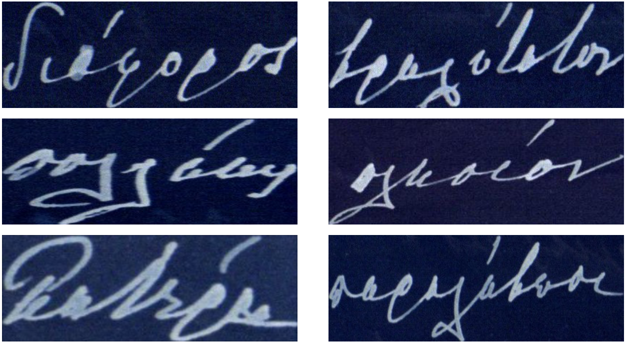


Figure: Word image samples from the “Memoirs” dataset.

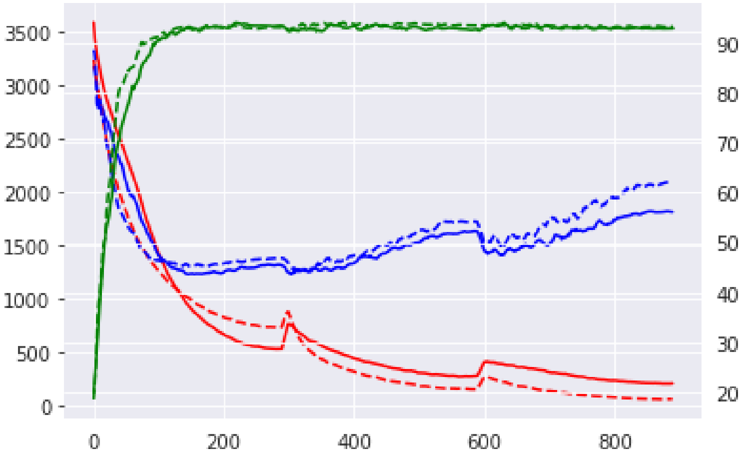


Figure: Results for the "PIOP-DAS" dataset (large models): Training set BCE loss, test set BCE loss and MAP are shown using red, blue and green colors respectively. Solid (dashed) lines correspond to quaternionic (real) models.

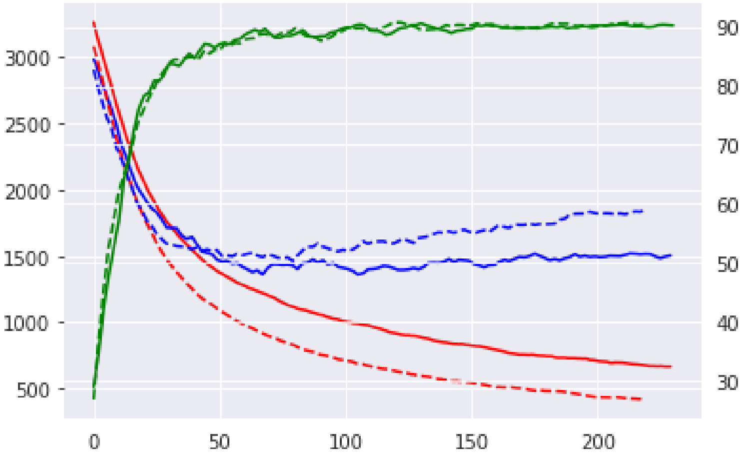
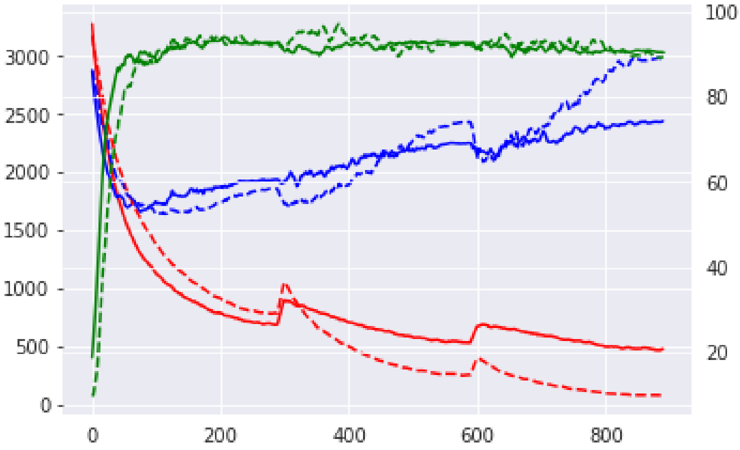


Figure: Results for the "PIOP-DAS" dataset (small models): Training set BCE loss, test set BCE loss and MAP are shown using red, blue and green colors respectively. Solid (dashed) lines correspond to quaternionic (real) models.



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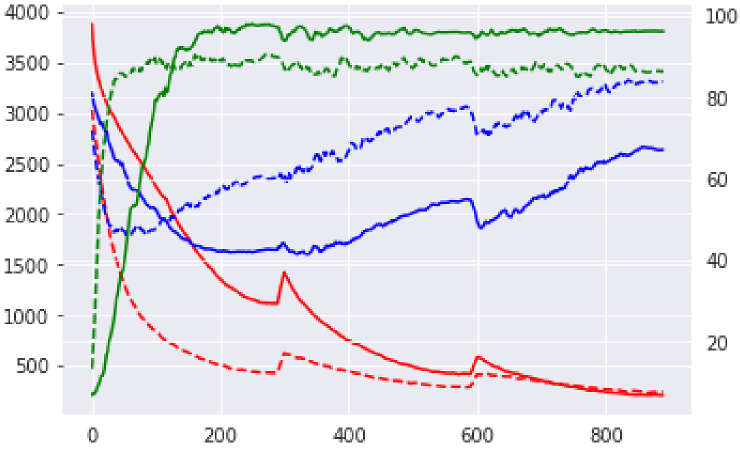

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Model accuracy comparison

Table: Model accuracy in terms of mean average precision (higher is better). Results correspond to the end of training and best figure reported over all epochs.

MAP%	Memoirs		PIOP-DAS	
	Last epoch	Best epoch	Last epoch	Best epoch
Quaternion / Standard	96.1%	98.6%	92.9%	94.3%
Real / Standard	89.3%	98.6%	93.3%	94.6%
Quaternion / Small	90.2%	94.6%	90.3%	92.8%
Real / Small	86.3	94.5%	90.0%	92.2%

Model size comparison

Table: Model size comparison, in terms of total number of trainable parameters. For the quaternionic variants, the equivalent number of real-valued parameters is reported, to ease comparisons.

Model / Network type	"Small" size	"Standard" size
Quaternionic Resnet	1,317,428	5,946,164
Real-valued Resnet	5,233,840	23,730,864

Summary

- The proposed Quaternionic ResNet works efficiently on the Keyword Spotting task..
- ..and **is on par with its real-valued counterpart.**
- Future work: Parameterized Hypercomplex layer
- Future work: Tests on more datasets, scripts, etc.

Thank you

Thank you for your attention! Questions?

Acknowledgments

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